The upper bound of the top mass and electroweak radiative corrections

Francesco Caravaglios

Scuola Normale superiore di Pisa and Infn sezione di Pisa

Abstract

We investigated the possibility of introducing sizeable negative corrections to the ε_{N1} ($\delta\rho$) parameter without affecting ε_{N3}. We have found that a proper vector-like family of fermions can imply such corrections. Differently from supersymmetry [12], this can be realized without introducing light particles easily observable at LEP II. Our example can be of particular interest if no new particle is found at LEP II and the ε_{N1} value is found to be small compared to the one expected in the case of a large top mass.

Introduction

In this paper, we study the radiative corrections of possible extensions of the standard model, paying particular attention to the ρ parameter which has relevant implications to the upper bound on the top mass. Precision tests are mainly relevant for two reasons [1-13]:

1) they could give signals of new physics;

2) their strong dependence on the top mass parameter gives an upper bound on this parameter.

Essentially, there are two measurements that provide us with an upper limit on the top mass: the direct and indirect [10] measurement of the Z_0 partial width into bottom and anti-bottom quarks (in other words ε_b [7]); and the ratio between the axial-vector coupling to leptons of the Z_0 and $W^\pm$ gauge bosons (ε_1 or $\delta\rho$ [2] [7]). The first one gives an upper limit (95% CL) on the top mass near above $210\ GeV$ [7] which can be violated only if new physics affects the Z_0-bottom vertex. This limit results from the present measurement of ε_b with an error of about 5 per mill, which is dominated by the error on $\alpha_s(M_Z^2)$. It is unlikely that this error will be lower than 4 per mill. So, except for large displacement of ε_b central value, this limit will not change much in the next future. On the contrary, the measurement of ε_1 combined possibly with a top mass measurement (for instance from the CDF experiment) could provide interesting new physics effects.

The experimental observation of the relation

$$\rho \simeq 1 \tag{1}$$

suggests that the symmetry breaking mechanism involves only Higgs doublets and that deviations from the above relation are obtained only through radiative corrections. Several authors have studied radiative corrections within and beyond the standard model [12] [11] [13] showing all possible consequence for the precision measurements of ε_1 both at low and high energy (LEP) experiments. As a general conclusion, within the models studied in the past, it is difficult to weaken the top mass upper bound and, on the contrary, it is easy to lower this upper bound. In supersymmetric models there is a way to weaken this limit [12], if some charginos are in the domain explorable at LEP II. Such particles would give, if just above the threshold of production at LEP I, a reduction of the Z-width ,affecting the ε_{N1} parameter through e_5 and raising the top mass upper limit. The price to pay is to introduce an analogous effect on the ε_{N3} parameter and the necessity of having some particles just above LEP I and easily observable at LEP II. The present experimental result for the ε_{N1} parameter is

$$\varepsilon_{N1} = 1.6 \pm 2.6 \times 10^{-3} \tag{2}$$

where the experimental error on ε_{N1} will be (probably) 1-1.5 per mill at the end of LEP I experiment. As an example, if the central value of this measure is not raised , we will be likely to have in the next future a statistically significant evidence for a negative contribution to ε_{N1}, in particular if a heavy top shows up ($M_{top} > 150\ (175)GeV$ implies $\varepsilon_{N1} > 3.5\ (5.8) \times 10^{-3}$ with a light Higgs).

In addition, if no new particle shows up at the LEP II phase[1] we will have to justify this anomalous negative correction on the ε_{N1} parameter. We show in this paper that a proper vector-family could lower in a sizeable way the $\delta\rho$ parameter without affecting ε_{N3}.

The vector-like family

The model we study is a very simple example of a vector-like family. As in SO(10) we consider a standard fermion family plus a right-handed neutrino [14], and we add a conjugate family where the role of left-handed and right-handed spinors are inverted with respect to the gauge interactions. We call

$$(u_L, d_L), (\nu_L, e_L), u_R, d_R, \nu_R, e_R \tag{3}$$

the first standard family[2] and

$$(\tilde{u}_R, \tilde{d}_R), (\tilde{\nu}_R, \tilde{e}_R), \tilde{u}_L, \tilde{d}_L, \tilde{\nu}_L, \tilde{e}_L. \tag{4}$$

the conjugate family. Fermions within parentheses are SU(2) doublets with isospin $(1/2, -1/2)$, while the others are singlets.

We allow only SU(2)×U(1) invariant mass terms and usual Yukawa interactions with a standard Higgs doublet, in order to have (1) at the tree level. Obviously it is possible to arrange the mass matrix between these fermions in order to have positive corrections both to ε_{N1} and ε_{N3}. On the contrary we are interested to have negative corrections to ε_{N1} without affecting ε_{N3}; in fact in this case the top mass upper bound is weakened. We assume a SU(2)×U(1) mass term between coloured particles so large (compared with breaking terms) that radiative corrections coming from this sector can safely be neglected. We introduce this requirement to simplify our calculations. Since every left-handed weyl spinor has its corresponding right-handed counterpart we can construct four leptonic Dirac fermions, one SU(2) doublet and two singlets, that we call

$$
\begin{aligned}
L &= (\nu_L + \tilde{\nu}_R, e_L + \tilde{e}_R) \\
\nu &= \nu_R + \tilde{\nu}_L \\
e &= e_R + \tilde{e}_L.
\end{aligned}
\tag{5}
$$

$$\tag{6}$$

Now we introduce a common invariant mass term also for the leptonic sector

$$\mathcal{L}_{inv} = m_0 \bar{L}L + m_0 \bar{\nu}\nu + m_0 \bar{e}e \tag{7}$$

and the yukawa interaction which will introduce a SU(2)×U(1) breaking mass term is

$$\mathcal{L}_{yuk} = \lambda_y (\bar{L}\phi\nu + \bar{L}\phi^c e + \bar{L}\phi\nu^c + h.c.) \tag{8}$$

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[1] In other words if no e_5 contribution can explain the ε_{N1} value [12].

[2]Hereafter, we call standard family the usual fermion family with the addition of a right-handed neutrino.

where the ν^c is the charge conjugated of ν. After symmetry breaking the Higgs acquires a *vev* $(v_0, 0)$ which introduces by (8) a mass matrix between fermions , which can be parametrized in terms of only one parameter $m = \lambda_y v_0$.

If we choose a mass basis we obtain the physical masses and the SU(2)×U(1) gauge interactions among the mass eigenstates. Both the lightest and the heaviest particle are neutral if $m < 2m_0/3$, and a discrete symmetry makes the lightest neutral particle stable[3].

The radiative corrections of the vector-like family

We focus our attention, now, on the radiative corrections induced by the model described above. Following the analysis described in [2][7] we only have to compute two quantities

$$e_1 = \frac{A_{33}(0) - A_{WW}(0)}{M_W^2} \tag{9}$$
$$e_3 = \frac{c}{s} F_{30}(0)$$

where $\Gamma_{ij} = -ig_{\mu\nu}(A_{ij}(0) + q^2 F_{ij}(q^2)) + q_\mu q_\nu(...)$ are the two point Green functions (one-particle irreducible as defined in [2]) and $i, j = 0, 3,$ w are the SU(2)×U(1) indices of the gauge bosons. The e_5 contribution [12] is negligible because we consider large fermion masses (well above the threshold of production at LEP I). We have neither vertex nor box diagrams. After the computation of the vacuum polarization of gauge bosons and taking the limit[4] $m << m_0$ we obtain

$$e_1 = -\frac{3\sqrt{2}}{10\pi^2} G_F \frac{m^4}{m_0^2} \tag{10}$$
$$e_3 = \frac{23}{60} \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{m^2}{m_0^2}. \tag{11}$$

We can see that, in the limit $m \to 0$ and m_0 constant (or equivalently $m_0 \to \infty$ and m constant) , our model gives no correction because it corresponds to restore the $SU(2) \times U(1)$ invariance of the mass matrix. As an example for the choice $m = 400 \ GeV$ and $m_0 = 1000 \ GeV$ (with this choice the lightest particle has $m_{light} = 200 \ GeV$, outside the range explorable by LEP II) we have the following numerical values

$$e_1 = -1. \times 10^{-2} \tag{12}$$
$$e_3 = 3. \times 10^{-4}. \tag{13}$$

It can be shown that if $M = M_0 + \delta M$ is the mass matrix among n Dirac fermions[5] $\psi = (\psi_1, ..., \psi_n)$ in a reducible or irreducible vector-like representation (n-dimensional), M_0 is

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[3] Possible implications to dark matter are beyond the scope of this paper.

[4]This relation is not necessary to make $\delta\rho < 0$ but we need it to make ε_{N3} small.

[5] The formula (14) can be generalized in the case of weyl fermions simply adjusting the factor in front of the trace.

a SU(2) invariant mass matrix (which gives equal mass to all fermions, for instance m_0) and δM is a SU(2) breaking matrix (with small elements compared with m_0, introduced by a Higgs doublet acquiring a *vev*) we obtain the general formula for ε_{N1}

$$\varepsilon_{N1} = \frac{\sqrt{2}}{5\pi^2} G_F \; Tr \; \frac{\left([M^2, T_3]^2 - [M^2, T_1]^2\right)}{m_0^2} + ... \tag{14}$$

where the dots stand for terms which are suppressed by factors of the type $\delta M_{i,j}^2/m_0^2$ which go to zero when $m_0 \to \infty$ and the symmetry breaking mass terms $\delta M_{i,j}$ remain constants; T_3, T_1 are the matrices of the gauge generators in our representation (T_3 corresponds to W_3 and T_1 corresponds to W_1). We can easily observe that, if

$$\left[M^2, T_3\right] = -\left[M^2, Y\right] = 0 \tag{15}$$

holds, than equation (14) gives[6] $\varepsilon_{N1} > 0$. So, the particular feature that in this case allows a negative contribution to ε_{N1} is the violation of (15). We remind the reader that we are considering the case of SU(2)×U(1) breaking mediated by Higgs doublets, which is strongly motivated by (1). In our simple model the mixing in the squared mass matrix between the ν_L and $\tilde{\nu}_R^c$ fermions (with respectively $T_3 = +1/2$ and $T_3 = -1/2$) introduces in the neutral sector of the representation a violation of equation (15). Alternatively one could introduce such violation in the charged sector but the resulting pattern of charged particles is less interesting[7]

Therefore, in conclusion, let us suppose that the top quark is heavy ($M_{top} > 150 \; GeV$) and that the ε_{N1}-value resulting at the end of LEP I program will remain small (for instance compatible with zero). Then we have to introduce a negative contribution to this parameter . One solution could be supersymmetry (through e_5) but if no particle shows up at LEP II then an SO(10) vector family, with a precise choice of masses, might be useful to solve that problem. Alternatively, if we believe that equation (15) holds, we argue that in a perturbative theory (with light Higgs) is difficult to evade the upper bound on the top mass coming from ε_{N1}.

Acknowledgements

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[6]We remind that the commutator is anti-hermitian and its square is definite negative.
[7] We have to introduce for example fundamental particles with charge > 1

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